

EXAM FOR HOMC 8

TEST 1

Q1. Suppose a, b are two numbers such that $a^2 + b^2 + 8a - 14b + 65 = 0$. Find the value of $a^2 + ab + b^2$.

- A. 37; B. 102; C. 5; D. 100; E. None of the above.

Q2. Find the remainder of 3^{2000} when it is divided by 13.

- A. 3. B. 6. C. 7. D. 5. E. None of the above.

Q3. Given triangle ABC , $AB = 8$, $BC = 7$, $CA = 6$. Extend BC to P such that $\Delta PAB \sim \Delta PCA$, then the length of PC is

- A. 7; B. 8; C.9; D. 10; E. 11.

Q4. How many the positive integer values of n such that $n^2 + 19n + 48$ is a perfect square number?

- A. 2. B.1 C.3. D. 0. E. None of the above.

Q5. The number of positive integer solutions (x, y, z) for the system of simultaneous equations

$$\begin{cases} xy + yz = 63 \\ xz + yz = 23 \end{cases} \text{ is}$$

- A. 1; B.2; C.3; D.4; E. None of the above.

Q6. Find the integer roots of equation $2^x - 3 = 65y$.

Q7. Find the value of expression $P = \frac{(4 \times 7 + 2)(6 \times 9 + 2)(8 \times 11 + 2) \cdots (100 \times 103 + 2)}{(5 \times 8 + 2)(7 \times 10 + 2)(9 \times 12 + 2) \cdots (99 \times 102 + 2)}$

Q8. Let \overline{abcdef} be a 6 – digit integer such that \overline{defabc} is 6 times the value of \overline{abcdef} . Find the value of $a + b + c + d + e + f$.

Q9. If $x^5 - 5qx + 4r$ is divisible by $(x - 2)^2$, find the values of q and r .

Q10. Let x, y are positive numbers such that $x + y = 1$. Find the minimum value of expression

$$P = \left(1 - \frac{1}{x^2}\right) \left(1 - \frac{1}{y^2}\right).$$

Q11. Suppose that triangle ABC has $AB > AC$, $AD \perp BC$ at D . P is an arbitrary point on AD different from A and D , prove that $PB - PC > AB - AC$.

Q12. Find the pair of integer numbers such that $x^2 + 2y^2 + 3xy + 3x + 5y = 15$.

Q13. Find the number of roots of equation $\left[\frac{x}{2}\right] + \left[\frac{2x}{3}\right] = x$

Q14. In the isosceles right triangle ABC , $AB = 1$, $A = 90^\circ$, E is midpoint of the leg AC . The point F is on the base BC such that $EF \perp BE$. Find the area of $\triangle CEF$.

Q15. If a, b, c denote the lengths of sides of a triangle, satisfying $a + b + c = 1$, prove that

$$a^2 + b^2 + c^2 + 4abc < \frac{1}{2}.$$